Name: Date:

Q01. Find the values of K so that the function f is continues at the indicated print.

$$F(x) = \begin{cases} \dfrac{k \cos x}{\pi - 2x} & ; \quad \text{if } x \neq \pi/2 \\ 2 & ; \quad \text{if } x = \pi \end{cases}$$

Q02. Differentiate the function $x^{\sin x} + (\sin x)^{\cos x}$

Q03. If $x = \sqrt{a^{\sin^{-1}t}}$, $y = \sqrt{a^{\cos^{-1}t}}$ show that $\dfrac{dy}{dx} = -\dfrac{y}{x}$

Q04. If $y = (\tan^{-1}x)^2$ show that $(x^2 + 1)^2 y_2 + 2x (x^2 + 1)y_1 = 2$

Q05. Verify Rolle's Theorem for the function $y = x^2 + 2$, $[-2 , 2]$

Q06. Differentiate $\sin^{-1}\left(\dfrac{4^{x+1}}{1 + 4^x}\right)$

Q07. Differentiate $\sin^2 x$ w.r.t $e^{\cos x}$.

Q08. If $x\sqrt{1 + y} + y\sqrt{1 + x} = 0$ prove that $\dfrac{dy}{dx} = \dfrac{1}{(1+x)^2}$

Q09. If $\cos y = x \cos (a + y)$ prove that $\dfrac{dy}{dx} = \dfrac{\cos^2 (a+y)}{\sin a}$

Q10. If $x = a (\cos t + t \sin t)$ and $y = a (\sin t - t\cos t)$. Find $\dfrac{d^2y}{dx^2}$

Q11. Find all points of discontinuity if $f(x) = \begin{cases} |x| + 3, & \text{if} \quad x < -3 \\ -2x, & \text{if } -3 < x < 3 \\ 6x + 2, & \text{if} \quad x > 3 \end{cases}$

Q12. Differentiate the function $\cos x^3 \sin^2 (x^2)$.

Q13. Find $\dfrac{dy}{dx}$. If $x^3 + x^2 y + xy^2 + y^3 = 81$

Q14. Differentiate $xy = e^{(x - y)}$

Q15. Find $\dfrac{dy}{dx}$ If $x = a (\cos t + t \sin t)$ and $y = a (\sin t - t \cos t)$.

Q16. If $y = 3 \cos (\log x) + 4 \sin (\log x)$. Show that $x_2 y_2 + xy_1 + y = 0$

Q17. Verify Rolle's Theorem for the function $y = x^2 + 2x - 8$, $[-4 , 2]$

Q18. Find $\dfrac{dy}{dx}$. $y = \tan^{-1}\left(\dfrac{\sin x}{1 + \cos x}\right)$

Q19. If $y = \begin{bmatrix} f(x) & g(x) & h(x) \\ l & m & n \\ a & b & c \end{bmatrix}$. Prove that $\dfrac{dy}{dx} = \begin{bmatrix} f'(x) & g'(x) & h'(x) \\ l & m & n \\ a & b & c \end{bmatrix}$

Q20. Find the value of K so that function is continuous. F(x) =

$$\begin{cases} kx + 1, & \text{if } x < \pi \\ \cos x, & \text{if } x > \pi \end{cases} \text{ is continuous at } x = \pi$$

Q21. Differentiate $y = 2\sqrt{\cot^2 x}$

Q22. Find $\frac{dy}{dx}$ if $\sin^2 y + \cos xy = \pi$

Q23. Find $\frac{dy}{dx}$ if $y^x + x^y + x^x = \alpha^b$

Q24. Find $\frac{dy}{dx}$ when $x = a(\theta - \sin\theta)$, $y = a(1+\cos\theta)$

Q25. If $y = 3e^{2x} + 2e^{3x}$. Prove that $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$

Q26. If $y = e^{a\cos^{-1}x}$. Show that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$.

Q27. If $(x - a)^2 + (y - b)^2 = c^2$. Prove $\dfrac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{2}}}{\frac{d^2y}{dx^2}}$ is a constant independent of a and b.

Q28. Find $\frac{dy}{dx}$, if $y = \sin^{-1}x + \sin^{-1}\sqrt{1 - x^2}$

Q29. $y = (\sin x - \cos x)^{(\sin x - \cos x)}$ Find $\frac{dy}{dx}$.

Q30. Find $\frac{dy}{dx}$, If $y = \frac{\sin(ax+b)}{\cos(cx+d)}$

Q31. Find $\frac{dy}{dx}$, If $y = \tan^{-1}\left(\frac{3x-x^3}{1- 3x^2}\right)$

Q32. Find $\frac{dy}{dx}$, If $y = (x \cos x)^x + (x \sin x)^{1/x}$

Q33. $x = a(\cos t + \log \tan\left(\frac{t}{2}\right))$, $y = a \sin t$, find $\frac{dy}{dx}$.

Q34. If $e^y(x + 1) = 1$. Show that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$

Q35. $y = a^{t + t/2}$ and $x = (t + \frac{t}{2})^a$. Find $\frac{dy}{dx}$.

Q36. Differentiate the following w.r.t. to x. $y = \cot^{-1}\left[\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right]$

Q37. If $y = \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x+....}}}$ Prove that $(1 -2y)\frac{dy}{dx} = \sin x$.

Q38. $\log \tan(\frac{\pi}{4} + \frac{x}{4})$. Show that $\frac{dy}{dx} - \sec x = 0$.

Q39. For what value of K is the following function continuous at x = 2?

$$F(x) = \begin{cases} 2x + 1 & ; x < 2 \\ k & ; x = 2 \\ 3x - 1 & ; x > 2 \end{cases}$$

Q40. Differentiate the following w.r.t. to $x^{\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]}$

Q41. If $y = \sin^{-1}\left[\frac{5x + 12\sqrt{1-x^2}}{13}\right]$ Find $\frac{dy}{dx}$.

Q42. Discuss the continuity of the following function at x = 0. $F(x) = \begin{cases} \frac{x^4 + 2x^3 + x^2}{\tan^{-1}x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

Q43. If $f(x) = \sqrt{\frac{\sec x - 1}{\sec x + 1}}$, find f'(x) also find $f'(\frac{\pi}{2})$

Q44. If $x^p y^q = (x + y)^{p+q}$ prove that $\frac{dy}{dx} = \frac{y}{x}$

Q45. If x = a sin pt, y = bcos pt find the value of $\frac{d^2y}{dx^2}$ t = 0.

Q46. If $y\sqrt{x^2 + 1} = \log(\sqrt{x^2 + 1} - x)$ prove that $(x^2 + 1)\frac{dy}{dx} + xy + 1 = 0$

Q47. If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \ldots + \infty}}}$. Prove that $\frac{dy}{dx} = \frac{\cos x}{2y - 1}$

Q48. If sin y = x sin(a + y) prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$.