CLASS XII - MATHEMATICS – CHAPTER 06

APPLICATION OF DERIVATIVES

Name: Date:

Q01. The length x of a rectangle is decreasing at the rate of 3 cm/mint and the width y is increasing at the rate of 2cm/min. when x = 10cm and y = 6cm, find the ratio of change of

(a). the perimeter (b). the area of the rectangle.

Q02. Find the interval in which the function of given by $f(x) = 4x^3 – 6x^2 – 72x + 30$ is

(a). strictly increasing (b). strictly decreasing.

Q03. Find point on the curve $\frac{x^2}{4} + \frac{y^2}{25} = 1$ at which the tangents are $\frac{x^2}{4}$

(a). parallel to x – axis (b). parallel to y – axis

Q04. The volume of a cube is increasing at a rate of $9cm^3s^{-1}$. How fast is the surface area increasing when the length of on edge is 10cm?

Q05. Find the interval in which the function is strictly increasing and decreasing. $(x+1)^3 (x-3)^3$

Q06. A ladder 5m long is leaning against a wall. The bottom of the ladder is pulled along the ground away from the wall, at the rate 2cm/s. how fast is its height on the wall decreasing when the foot of the ladder is 4m away from the wall.

Q07. A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the y-coordinate is changing 8 times as fast as the x – coordinate.

Q08. Find the interval in which increase/decrease. f(x) = sin 3x , 0, x ε $[0,\frac{\pi}{2}]$

Q09. Find the intervals in which the function f given by f (x) = sin x + cos x, $0 \leq x \leq 2\pi$ is strictly increasing or decreasing.

Q10. Sand is pouring from a pipe at the rate of $12cm^3/s$. the falling sand forms a cone on the ground in much a way that the height of the cone is always one – sixth of the radius of the here. How fast is the height of the sand cone increasing when the height in 4cm.

Q11. The total revenue in RS received from the sale of x units of the product is given by $R(x) = 13x^2 + 26x + 15$ find MR when 17 unit are produce.

Q12. Prove that 4sinθ / (2 + cosθ) – θ is an increasing function = x of θ in $[0, \frac{\pi}{2}]$.

Q13. Prove that the function of given by f(x) = log (sin x) is strictly increasing on $[0, \frac{\pi}{2}]$ and strictly decreasing on $[\frac{\pi}{2}, \pi]$

Q14. A balloon, which always remains spherical on inflation, is being inflated by pumping in $900cm^3/s$. find the rate at which the radius of the balloon increase when the radius is 15cm.

Q15. A circular disc of radius 3cm is being heated. Due to expansion, their radius increase at the rate of 0.05 cm/s. find the rate at which its area is increasing when radius is 3.2cm.

Q16. Find the intervals in which the function f given by $\frac{4\sin x - 2x - x\cos x}{2 + \cos x}$ is

(a). increasing (b). decreasing

Q17. Find the interval in which the function f given by $f(x) = x^3 + \frac{1}{x^3}, x \neq 0$ is

(a). increasing (b). decreasing.

Q18. Find the equation of the normal to the curve $x^2 = 4y$ which passes through the point (1, 2).

Q19. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$.

Q20. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.

Q21. The two equal side of an isosceles Δ with fixed base b are decreasing at the rate of 3cm/s. How fast is the area decreasing when the two equal sides are equal to the base?

Q22. A men of height 2m walks at a uniform speed of 5km/h away from a lamp, past which is 6m high. Find the rate at which the lengths of his shadow increase.

Q23. A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lower most. Its semi vertical angle is $\tan^{-1}(0.5)$ water is poured into it at a constant rate of $5cm^3/hr$. Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 4m.

Q24. Find the interval in which the function given by $f(x) = \frac{3x^4}{10} - \frac{4x^3}{5} - 3x^2 + \frac{36x}{5} + 11$ is

(a). Strictly increasing (b). Strictly decreasing

Q25. Show that $f(x) = \tan^{-1}(\sin x + \cos x)$ is always an increasing function in $[0, \frac{\pi}{4}]$.

Q26. A wire of length 28m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the length of the two pieces to that the combined areas of the square and the circle is minimum.

Q27. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

Q28. Show that the right circular cone of least curved surface area and given volume has an altitude equal to $\sqrt{2}$ time the radius of the base.

Q29. Show that semi–vertical angle of right circular cone of given surface area and maximum volume is $\sin^{-1}(\frac{1}{3})$

Q30. A square piece of tin of side 18cm is to be made into a box without top by cutting a square from each corner and folding of the flaps to form the box. What should be the side of the square to be cut off so that the volume of the box is the maximum possible?

Q31. Show that the right circular cylinder of given surface and maximum volume is much that its height is equal to the diameter of the base.

Q32. Show that the height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and having semi-vertical angle is one third that of the cone and the greatest volume of cylinder is $\frac{4\pi^3 h \tan\alpha}{27}$

Q33. Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.

Q34. A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2m and volume is $8m^3$. If building of tank costs Rs 70 per sq. metres for the base and Rs 45 per sq. metres for sides what is the cost of least expansive tank?

Q35. The sum of the perimeter of a circle and square is k, where K is some constant. Prove that the sum of their area is least when the side of square is double the radius of circle.

Q36. A window is the form of a rectangle surmounted by a semi circular opening the total perimeter of the window is 10m. Find the dimensions of the window to admit maximum light through the whole opening.

Q37. A point on the hypotenuse of a triangle is at distance a and b from the sides of the triangle. Show that the minimum length of the hypotenuse is $(a^{\frac{2}{3}} + b^{\frac{2}{3}})^{\frac{3}{2}}$.